

# Nuclear Structure of the Samarium Isotopes $^{152-154}\text{Sm}$ Using Models of IBM-2 and DDM

Saad N. Abood<sup>1</sup> Laith A. Najim<sup>2\*</sup>, Jundi Kh. Yousif<sup>3</sup>

<sup>1</sup>Department of Physics, College of Science, AL-Nahrain, University, Baghdad, IRAQ

<sup>2</sup>Department of Physics, College of Science, Mosul University, Mosul, IRAQ

<sup>3</sup>Department of Physics, Faculty of Science, Zakho University, Duhok, IRAQ

\*E-mail: dr\_laithahmed@yahoo.com

**Abstract-** Samarium isotopes ( $Z=62$ ) lie in the traditional rotational to transitional-spherical region that occurs at the range of deformed nuclei. Energy levels, Electromagnetic transitions  $B(E2)$ ,  $B(M1)$ , mixing ratio  $\delta(E2/M1)$  and monopole transitions  $B(E0)$  for the selected transitions in  $^{152-154}\text{Sm}$  are calculated in the frame work of collective models proton-neutron Interacting boson model (IBM-2) and Dynamic deformation model (DDM). The results obtained for  $^{152-154}\text{Sm}$  are reasonably in a good agreement with the known experimental results.

**Keywords-** Interacting boson model; dynamic deformation model; electromagnetic transition probabilities; mixing ratios; electric monopole transitions.

## I. INTRODUCTION

Accurate information about the multipole moments of the low-lying excited states of  $^{152}\text{Sm}$  and  $^{154}\text{Sm}$  isotopes is of a particular interest, since these nuclei are in the transitional region between spherical and deformed nuclei. These spectra of states of the transitional nuclei fit neither a vibrational nor a rotational pattern, although they exhibit some characteristics of both. The low-lying excited states of  $^{152}\text{Sm}$  and  $^{154}\text{Sm}$  isotopes can be classified into ground, beta and gamma bands [1].

The phase transition from spherical to deformed shape which takes place in Sm isotopes has stimulated many authors to study theoretically and experimentally the area of these isotopes. The transition occurs between  $^{150}\text{Sm}$  and  $^{152}\text{Sm}$ . The former is a vibrational-like isotope while the latter is a rotational like one.

The isotopes  $Z=62$  and  $N=90$  to  $92$  are well abrupt changes in nuclear properties between almost spherical in  $N=82$  to well deformed in  $N = 92$  [2]. So Sm isotopes have provided a useful testing ground for nuclear structure calculations. Scholten *et al.*, [3] showed that the Sm isotopes lie in a transitional region from vibrational  $U(5)$  to the rotational limit  $SU(3)$  of the IBM.

Moinester *et al.*, [4] used the interacting boson model (IBM) to interpret the ground state charge distributions and lowest  $2^+$  transition charge densities of the even  $^{144-154}\text{Sm}$ . Also, Castanos [5] has been used the effective IBM Hamiltonian to describe the low-lying energy spectra of several series of even-even isotopes. Applications are made to the Xe, Ba, Sm, Gd and U isotopes. In each case a single effective IBMA Hamiltonian containing at most six parameters reproduces some 50 experimental energies with an root mean square deviation of about 0.1MeV or less.

Diab [6] studied the electric monopole transitions and structure of  $^{150}\text{Sm}$  in IBM-2, and showed that the contour plot of the potential energy surfaces  $V(\beta,\gamma)$  shows that  $^{150}\text{Sm}$  is a spherical nucleus and has vibrational characters.

The aim of this work is to study the nuclear structure and electromagnetic transitions in  $^{152-154}\text{Sm}$  isotopes within framework of IBM-2 and DDM and to compare the results with the available experimental data. In this work, predictions of the IBM-2 and DDM for the energy levels, electromagnetic transitions probabilities  $B(E2)$  and  $B(M1)$ , multipole mixing ratios and monopole matrix elements in  $^{152-154}\text{Sm}$  isotopes are presented.

## II. The Models

### Interacting Boson Model

The Interacting Boson Model (IBM) [7-8] has been remarkably successful in describing the low-lying collective states in many medium to heavy even-even nuclei. The neutron-proton version of the interacting boson model (IBM-2), later suggested by Iachello and Arima [9] who distinguish between neutron ( $\nu$ ) and proton ( $\pi$ ) boson, is used in the present work, and a full description of the IBM-2 is found in ref. [10]. The Hamiltonian operator in IBM-2, which has been used to calculate the energy level and hence the gamma transitions matrix elements, has three parts, the first part is for proton bosons, the second is for neutron bosons whereas the last one is used to describe the interaction between different bosons:

$$H = H_{\pi} + H_{\nu} + V_{\pi\nu} \quad (1)$$

The Hamiltonian generally used in the phenomenological calculations can be written as:

$$H = \varepsilon_d (n_{d\nu} + n_{d\pi}) + \kappa(Q_{\nu} \cdot Q_{\pi}) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi} \quad (2)$$

Where, the dot denotes the scalar product. The first term represents the single-boson energy for neutron and proton,  $\varepsilon_d$  is the energy difference between  $s$ - and  $d$ - boson and  $n_{d\rho}$  is the number of  $d$ -bosons, where  $\rho$  corresponds to  $\pi$  (proton) or  $\nu$  (neutron) bosons. The second term denotes the main part of the boson-boson interaction, i.e., the quadrupole-quadrupole interaction between neutron and proton bosons with the strength  $\kappa$ . The quadrupole operator is

$$Q_{\rho} = [d_{\rho}^{+} s_{\rho} + s_{\rho}^{+} d_{\rho}]^{(2)} + \chi_{\rho} [d_{\rho}^{+} d_{\rho}]^{(2)} \quad (3)$$

Where,  $\chi_{\rho}$  determines the structure of the quadrupole operator and is determined empirically. The square brackets in eq. (3) denote the angular momentum coupling.

The terms  $V_{\pi\pi}$  and  $V_{\nu\nu}$ , in eq.(2) correspond to the interaction between similar bosons, and there are sometimes included in order to improve its fits to experimental energy spectra. They are of the form:

$$V_{\rho\rho} = \frac{1}{2} \sum_{L=0,2,4} C_L^{\rho} ([d_{\rho}^{+} d_{\rho}^{+}]^{(L)} \cdot [d_{\rho} d_{\rho}]^{(L)}). \quad (4)$$

However, their effects are usually considered minor and often neglected [11].

The Majorana term,  $M_{\pi\nu}$ , shifts the states with mixed proton-neutron symmetry with respect to the totally symmetric ones. Since the little experimental information is known about such states with mixed symmetry, we did not attempt to fit the parameters appearing in eq. (2), but rather took constant values for the  $^{152-154}\text{Sm}$  isotopes (which contains three parameters  $\xi_1, \xi_2$  and  $\xi_3$ ) may be written as:

$$M_{\pi\nu} = \frac{1}{2} \xi_2 ([s_{\nu}^{+} d_{\pi}^{+} - d_{\nu}^{+} s_{\pi}^{+}]^{(2)} \cdot [s_{\nu} d_{\pi} - d_{\nu} s_{\pi}]^{(2)}) - \sum_{k=1,3} \xi_k ([d_{\nu}^{+} d_{\pi}^{+}]^{(k)} \cdot [d_{\nu} d_{\pi}]^{(k)}). \quad (5)$$

In this work, we give the Hamiltonian of IBM-2 and DDM terms of the formalism; and study the nuclear structures of  $^{152-154}\text{Sm}$  isotopes by these Hamiltonians.

### Dynamic Deformation Model (DDM)

The dynamic deformation model has been developed over many years starting from the Paring Plus Quadrupole model (PPQ) of Kumar and Baranger [12-13]. The DDM is an ambitious attempt to the collective spherical-transitional-deformed transitions and to span from the  $s$ - $d$  shell to heavy nuclei using a microscopic theory of collective motion. No fitting parameters are required to obtain the data for a particular nucleus.

The detailed formalism and early results may be found in Kumar *et al.*, [14] and Kumar [15]. Here we give briefly the main aspects of the model. The theory can be divided into two main parts: a microscopic derivation of a collective Hamiltonian, and a numerical solution of the Hamiltonian. The microscopic Hamiltonian is composed of a demoralized Nilsson-type single particle plus pairing and has the form:

$$H = H_{av} + V_{res} \quad (6)$$

Where,

$$H_{av} = \frac{p^2}{2M} + \frac{1}{2} M \sum_{k=1}^3 \omega_k^2 \chi_k^2 + \hbar \omega_0 [v_{ls} l \cdot s + v_{ll} (l^2 - \langle l^2 \rangle_N)] \quad (7)$$

Combining all the various contributions together, the potential energy is written as:

$$V_{coll} = V_{DM} + \delta U + \delta V_{proj} + \delta E_{pair} \quad (8)$$

Where  $\delta V_{proj}$  is a nine-dimensional projection correction introduced by Kumar [15]. The generalized cranking method is employed to derive the general expression for mass parameters  $B_{\mu\nu}(\beta, \gamma)$  as used in the collective kinetic energy which can be written as:

$$T_{coll} = \frac{1}{2} \sum_{\mu\nu} B_{\mu\nu} \alpha_\mu \alpha_\nu^* \quad (9)$$

This kinetic energy function is quantized by Pauli method. The DDM code used for our calculation is a modified version of the latest DDM code which was developed for superheavy nuclei. The single particle levels and the configuration space ( $n = 0$  to 8) employed in the present calculation, as well as the deformation definition, are identical to those of [14].

### III. RESULTS AND DISCUSSIONS

#### Energy Levels

The isotopes chosen in this work are A=152, 154 due to the present of the experimental data for the energy levels. We have  $N_\pi = 6$ , (12 protons outside the closed shell 50), and  $N_\nu$  varies from 4 for  $^{152}\text{Sm}$  to 5 for  $^{154}\text{Sm}$ , measured from the closed shell at N=82. While the parameters  $\kappa, \chi_\rho$ , and  $\varepsilon_\rho$ , as well as the Majorana parameters  $\xi_k$ , with  $k=1,2,3$  were treated as free parameters and their values were estimated by fitting with the experimental values. In the calculation of  $\delta(E2/M1)$ , it is found that there is a great effect of the Majorano parameter  $\xi_2$  on the value and sign of E2 and M1 matrix elements. The procedure was made by selecting the traditional value of the parameters and allowing one parameter to vary while

keeping the others constant until the best fit with the experiment is obtained. This was carried out until one overall fit was obtained. The best values for the Hamiltonian parameters of IBM-2 are given in table I. In the DDM there is no parameter fitted in the Hamiltonian except for Z (atomic number) and A (mass number).

Table I  
IBM-2 Hamiltonian parameters, all parameters in MeV units

Isotopes	$\varepsilon$	$\kappa$	$\chi_\nu$	$\chi_\pi$	$\xi_2$	$\xi_3$	$\nu = 0,2,4$	$\nu = 0,2,4$
$^{152}\text{Sm}$	0.	-	-	-	0.12	0	-0.5, 0.4,	-0.5, 0.4,
	4	0.	0.	1.		.	-0.8	-0.8
	3	02	8	2		1		
$^{154}\text{Sm}$	0.	-	-	-	0.12	0	-0.5, 0.4,	-0.5, 0.4,
	3	0.	0.	1.		.	-0.8	-0.8
	4	03	8	2		1		
							9	

A Concentration was made on the  $2_1^+$  to make a reasonable fits to the experimental data. A sample of experimental and theoretical values of energy levels is taken from table II. It is noticed that a good agreement was obtained for the gamma and beta bands for  $^{152}\text{Sm}$ . In addition, table II shows a comparison between the experimental and theoretical energy levels of the ground band in  $^{152-154}\text{Sm}$  isotopes. There is also an agreement between  $2_1$  and  $4_1$ , but the DDM model is not able to predict the  $6_1$  and this may be due to the high spin of this state. Actually this has slim effects on the calculations of transitions probability.

The experimental energy ratio between first and second excited state is  $R_{4/2} = E(4_1^+) / E(2_1^+)$  which has limiting values of 2 for quadrupole vibrator, 2.5 for gamma-soft and 3.33 for an ideal symmetric rotor. The ratio  $R_{4/2}$  for  $^{152}\text{Sm}$  equals 3.0004 for the experimental data, 3.048 for IBM-2, and 3.304 for DDM results. The ratio  $R_{4/2}$  for  $^{154}\text{Sm}$  equal 3.256 for experimental data, 3.191 for IBM-2 and 3.154 for DDM. Therefore, from these ratio values, we deduced that the considered nuclei lie in the transitional region from gamma-soft a rotational

shape O(6)- SU(3), and nearly close to the rotational limit.

Table II

A comparison between the experimental and calculated energy levels for  $^{152-154}\text{Sm}$  in ( MeV unit).

$J_i^+$	$^{152}\text{Sm}$			$^{154}\text{Sm}$		
	Exp. [16]	IBM-2	DDM	Exp. [16]	IBM-2	DDM
0(1)	0.0	0.0	0.0	0.0	0.0	0.0
2(1)	0.1218	0.123	0.114	0.082	0.0821	0.084
4(1)	0.366	0.375	0.387	0.267	0.262	0.265
0(2)	0.685	0.714	0.715	1.100	1.153	1.057
6(1)	0.706	0.711	0.585	0.549	0.602	0.631
2(2)	0.811	0.952	0.855	1.178	1.213	1.192
4(2)	1.023	1.001	1.039	1.371	1.421	1.430
2(3)	1.086	1.272	1.397	1.440	1.521	1.438
3(1)	1.234	1.003	1.559	1.539	1.557	1.541
4(3)	1.372	1.251	1.705	1.660	1.783	1.664
5(1)	-	1.432	1.888	-	1.980	2.321
6(2)	1.311	1.414	1.312	-	1.450	2.365
0(3)	1.080	1.139	1.473	1.202	1.201	1.450
2(4)	1.239	1.593	1.893	-	1.621	2.540
4(4)	-	2.211	2.210	1.662	1.783	1.982
8(1)	1.1250	1.290	1.254	0.903	1.021	0.973

*Electric quadrupole Transition Probability*

In IBM-2, the E2 , transition operator is given by:

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu \quad (10)$$

Where  $Q_\rho$  is the same as in eq.(3) ,  $e_\pi$  and  $e_\nu$  are boson effective charges depending on the boson number  $N_\rho$  ( $\rho = \pi$  or  $\nu$ ) and they can take any value to fit the experimental results ( $B(E2;2_1^+ \rightarrow 0_1^+)$ ). The method is explained in ref. [17], and the effective charges calculated by this method for  $^{152-154}\text{Sm}$  isotopes were  $e_\nu = 0.10eb$  and  $e_\pi = 0.13eb$ . Table III gives the electric transition probability B(E2).

The  $B(E2;2_1^+ \rightarrow 0_1^+)$  &  $B(E2;4_1^+ \rightarrow 2_1^+)$  values increased as neutron number increases toward the middle of the shell while the value of  $B(E2;2_2^+ \rightarrow 2_1^+)$  has a small value

because it contain mixtures of M1. The value of  $B(E2;2_2^+ \rightarrow 0_1^+)$  is small because this transition is forbidden (from quasibeta band to ground state band) whereas the values of IBM-2 and DDM are in a good agreement with the available experimental data.

In table III, the electric quadrupole moment  $Q(2_1^+)$  for the first excited state in  $^{152-154}\text{Sm}$  isotopes are well described indicating that the nucleus  $^{152}\text{Sm}$  has a prolate shape in its first excited state.

Table III

Electric Transition probability  $B(E2)$  for  $^{152-154}\text{Sm}$  isotopes in(  $e^2b^2$  units)

$J_i^+ \rightarrow J_f^+$	$^{152}\text{Sm}$			$^{154}\text{Sm}$		
	Exp.(*)	IB M-2	DD M	Exp.	IBM -2	DD M
$2_1 \rightarrow 0_1$	0.670(15)	0.684	0.360	0.922(40)	0.913	0.940
$4_1 \rightarrow 2_1$	1.017(4)	0.980	0.932	1.186(39)	1.231	1.400
$6_1 \rightarrow 4_1$	1.179(33)	1.003	1.092	1.374(47)	1.393	1.350
$0_2 \rightarrow 2_1$	0.176(11)	0.120	0.454	0.235	0.250	0.235
$2_2 \rightarrow 0_1$	0.00456(34)	0.007	0.0252	0.060(14)	0.0071	0.013
$2_2 \rightarrow 2_1$	0.0258(26)	0.025	0.0752	0.012	0.014	1.970
$2_2 \rightarrow 4_1$	0.091(11)	0.077	0.165	0.024	0.028	0.054
$4_2 \rightarrow 2_1$	0.0035(35)	0.001	0.0049	-	0.027	0.012
$4_2 \rightarrow 4_1$	0.037(23)	0.041	0.0545	-	-	-
$2_3 \rightarrow 0_1$	0.0163(11)	0.027	0.662	0.013(3)	0.015	0.033
$2_3 \rightarrow 2_1$	0.0417(42)	0.051	0.1012	0.02	0.022	0.047
$2_3 \rightarrow 4_1$	0.0416(32)	0.003	-	-	0.0009	0.010
$4_3 \rightarrow 2_1$	0.0035(13)	0.008	0.0228	-	-	-
$4_3 \rightarrow 4_1$	0.037(13)	0.049	0.0211	-	-	-
Q(2 <sub>1</sub> )	-1.8(0.6)	-	-	-	-	-

		1.7 65	1.64			
--	--	-----------	------	--	--	--

\* Experimental data are taken from references [16, 18,19 ,20,21]

### Magnetic Transition Probability (BMI)

After calculating E2 matrix elements we look after elements as follows:

The M1 operator is obtained by letting  $l = 1$  in the single boson operator of the IBM-2 and can be written as:

$$T^{(M1)} = \left[ \frac{3}{4\pi} \right]^{1/2} (g_\pi L_\pi^{(1)} + g_\nu L_\nu^{(1)}) \quad (11)$$

Where  $g_\pi, g_\nu$  are the boson g-factors in units of  $\mu_N$  and  $L^{(1)} = \sqrt{10}(d^+ x \tilde{d})^{(1)}$ . This operator can be written as:

$$T^{(M1)} = \left[ \frac{3}{4\pi} \right]^{1/2} \left[ \frac{1}{2}(g_\pi + g_\nu)(L_\pi^{(1)} + L_\nu^{(1)}) + \frac{1}{2}(g_\pi - g_\nu)(L_\pi^{(1)} - L_\nu^{(1)}) \right] \quad (12)$$

The first term on the right hand side of Eq.(12), is diagonal, and therefore, M1 transitions can be written as:

$$T^{(M1)} = 0.77 \left[ (d^+ \tilde{d})_\pi^{(1)} - (d^+ \tilde{d})_\nu^{(1)} \right] (g_\pi - g_\nu) \quad (13)$$

The direct measurement of B(M1) matrix elements is normally difficult, so the M1 strength of gamma transition may be expressed in terms of the multipole mixing ratio which can be written as [22]

$$\delta(E2/M1) = 0.835 E_\gamma (MeV) \frac{\langle J_f | T(E2) | J_i \rangle}{\langle J_f | T(M1) | J_i \rangle} \quad \text{In eb/}\mu_N \quad (14)$$

Having fitted E2 matrix elements, one can use them to obtain M1 matrix elements and then the mixing ratio  $\delta(E2/M1)$ , compare them with the prediction of the model using the operator in (eq.9). The  $g_\pi$  and  $g_\nu$  have to be estimated, if they were not measured in the case of Sm isotopes. The g factors

may be estimated from the experimental magnetic ( $\mu$ ) moment of the  $2^+_1$  state ( $\mu=2g$ ). In the phenomenological studies  $g_\pi$  and  $g_\nu$  are treated as parameter, and kept constant for a whole isotope chain. The total g factor is defined by Sambataro *et. al.*, [23] as:

$$g = g_\pi \frac{N_\pi}{N_\pi + N_\nu} + g_\nu \frac{N_\nu}{N_\pi + N_\nu} \quad (15)$$

Many relations could be obtained for a certain mass region and then the average  $g_\pi$  and  $g_\nu$  values for this region could be calculated. One of the experimental B(M1) and the relation above has been used to find out that  $g_\pi - g_\nu = 0.53 \mu_N$ . The estimated values of the parameter are  $g_\pi = 0.84 \mu_N$  and  $g_\nu = 0.31 \mu_N$ . These were used to calculate the mixing ratio  $\delta(E2/M1)$ . The ratios were calculated for some selected transitions and listed with the available experimental data in table IV. A good agreement is obtained between the calculated value by (IBM-2 and DDM) and the experimental data in sign and magnitude.

Table IV  
Mixing Ratios for  $^{152-154}\text{Sm}$

$J_i^+ \rightarrow J_f^+$	$^{152}\text{Sm}$			$^{154}\text{Sm}$		
	Exp. [[16,22]	IB M-2	DD M	Exp. [16,22]	IBM-2	DDM
$2_2 \rightarrow 2_1$	$8_{-3}^{+9}$	9.5 0	10.8	$56_{-25}^{+130}$	34	0.054
$4_2 \rightarrow 4_1$	3(1)	5.1 02	4.0	-1.1	- 0.97 0	0.320
$2_3 \rightarrow 2_1$	$-11_{-0.8}^{+0.7}$	- 9.5 21	- 24.3	$0.8_{-6}^{+15}$	0. 620	0.047
$3_1 \rightarrow 2_1$	$-30_{-12}^{+7}$	-8.0	- 26.7	-7.5	-5.22	2.40
$3_1 \rightarrow 4_1$	$-12.2_{-0}^{+1}$	- 11. 32	- 16.8	-	0.03 2	0.052
$2_3 \rightarrow 2_2$	-	6.6	19.6	-	0.00 7	0.003

$4_3 \rightarrow 4_1$	$-3_{-2.4}^{+1}$	- 2.8 1	-9.6	-	0.04 21	0.021
$3_1 \rightarrow 2_2$	-	7.4 1	6.9	-	0.07 31	0.098 2
$3_1 \rightarrow 4_2$	-	- 7.5 1	- 21.0	-	1.2* $10^{-4}$	2.73* $10^{-4}$
$4_3 \rightarrow 4_2$	-	4.4 62	9.70	-	2.5* $10^{-3}$	3.72* $10^{-3}$

The magnetic dipole transition probability is presented in table V. It is worth mentioning here that there is no experimental data to compare with the theoretical results. The  $B(M1; 2_3^+ \rightarrow 2_1^+)$  values decrease for two models, implying some collective effects. The large B(M1) values in IBM-2 are due to the F-spin vector character of  $2_3^+$  state in  $^{152-154}\text{Sm}$ . The  $B(M1; 0_1^+ \rightarrow 1_1^+)$  is still sizable in  $^{152-154}\text{Sm}$  (increased with the increase of neutron number) because of the transition from ground state band to mixed symmetry state  $1_1^+$  in IBM-2 as well as in DDM.

Table V  
Magnetic Transition probability  $B(M1)$  for  $^{152-154}\text{Sm}$  isotopes in ( $\mu_N^2$  units)

$J_i^+ \rightarrow J_f^+$	$^{152}\text{Sm}$		$^{154}\text{Sm}$	
	IBM-2	DDM	IBM-2	DDM
$2_2 \rightarrow 2_1$	0.07	$0.4 * 10^{-2}$	0.007	$4.081 * 10^{-4}$
$2_3 \rightarrow 2_1$	0.06	$0.282 * 10^{-2}$	0.020	$1.055 * 10^{-3}$
$0_1 \rightarrow 1_1$	1.460	1.367	1.810	$1.939 * 10^{-6}$
$3_1 \rightarrow 2_1$	0.262	$0.082 * 10^{-2}$	-	-
$3_1 \rightarrow 2_2$	0.082	$0.727 * 10^{-2}$	-	-
$3_1 \rightarrow 3_2$	0.087	$0.327 * 10^{-2}$	-	-

#### Monopole Transition Probability $B(E0)$

Monopole transitions (E0) are known to be pure penetration effect, where the transition is caused by an electromagnetic interaction between the nuclear charge and the atomic electron penetrating the nucleus. An E0 transition occurs between two states of the same spin and parity by transferring the energy and zero unit of angular momentum. Thus E0 has no

competing gamma ray. These transitions are different from zero only in the case where the transition is accompanied by the nucleus surface change, for example in the nuclear models where the surface is assumed to be fixed E0 transitions are strictly forbidden. Electric monopole transitions can occur not only in  $0^+ \rightarrow 0^+$  transition but also, in competition with gamma multipole transition and depending on transition selection rules may compete in any  $\Delta I = 0$  decay such as  $2^+ \rightarrow 2^+$ . At transitions energies greater than  $2m_0c^2$ , monopole pair production is also possible.

The monopole transition operator T0 is given by [24]

$$T(E0) = \beta_{0\pi} d_{\pi}^+ \cdot d_{\pi}^- + \beta_{0\nu} d_{\nu}^+ \cdot d_{\nu}^- + \gamma_{0\nu} N_{\pi} + \gamma_{0\nu} N_{\nu}$$

E0 reduced transitions probability is written as in IBM-2 [24]

$$B(E0; J_i - J_f) = e^2 R^4 \rho^2(E0) \quad J_i = J_f \quad (16)$$

Where e is the electronic effective charge, R is the nuclear radius and  $\rho(E0)$  is the transition matrix element. However, there are only limited cases where  $\rho(E0)$  can be measured directly. In most cases, we have to determine the intensity ratio of E0 to the competing E2 transition calling this as  $X(E0/E2)$  value [25] which can be written as

$$X(E0/E2) = \frac{B(E0; J_i - J_f)}{B(E2; J_i - J_f)} \quad (17)$$

Where,  $J_f = J_{f'}$  for  $J_i \neq 0$ , and  $J_f = 0, J_{f'} = 2$  for  $J_i = 0$ .

The  $T^{(E0)}$  operator may be found by setting  $l = 0$  on the IBM-2 operator [26]

$$\rho_{if}(E0) = \frac{Z}{R_0^2} \sum \tilde{\beta}_{0\rho} \langle f | d_{\rho}^+ x d_{\rho} | i \rangle \quad (18)$$

Where  $R_0 = 1.2A^{1/3}$  fm and  $\rho(E0)$  is a dimensionless quantity. The two parameters  $\tilde{\beta}_{0\pi}, \tilde{\beta}_{0\nu}$  in equation

(14) may be estimated by fitting in isotope shift, which is the difference in the square radius  $\delta\langle r^2 \rangle$  between the neighboring isotopes in their ground state [27]

i.e.

$$\delta\langle r^2 \rangle = \langle e|r^2|e \rangle - \langle 0_1^+|r^2|0_1^+ \rangle$$

$$\delta\langle r^2 \rangle = \beta_{0n}\delta n_{d\pi} + \beta_{0v}\delta n_{dv} \quad (19)$$

where  $\delta n_{d\rho} = \langle e|d_\rho^+ d_\rho^-|e \rangle - \langle 0|d_\rho^+ d_\rho^-|0 \rangle$

In the case of Samarium isotopes, the measured isotopes shift [28] was used to find the parameters used in the IBM-2 calculations and they are  $\tilde{\beta}_\pi = 0.045 fm^2$  and  $\tilde{\beta}_v = 0.031 fm^2$  which produced the monopole matrix elements. The calculated and experimental values of  $\rho(E0)$ ,  $X(E0/E2)$ , and the isomer shifts are given in tables VI, VII and VIII respectively.

Table VI  
Monopole matrix element  $\rho(E0)$  for  $^{152-154}Sm$

$J_i^+ \rightarrow J_f^+$	$^{152}Sm$			$^{154}Sm$	
	Exp. [ 23 ]	IBM-2	DDM	IBM-2	DDM
$0_2 \rightarrow 0_1$	0.26(2)	0.311	0.365	0.432	0.563
$2_2 \rightarrow 2_1$	0.255(12)	0.402	0.363	0.532	0.672
$2_3 \rightarrow 2_1$	-	0.032	0.007	0.021	0.0117
$0_3 \rightarrow 0_1$	-	0.021	0.038	0.0371	0.0627
$0_3 \rightarrow 0_2$	0.261(36)	0.231	0.442	0.281	0.376

Table VII  
 $X(E0/E2)$  for  $^{152-154}Sm$

$J_i^+ \rightarrow J_f^+$	$^{152}Sm$			$^{154}Sm$	
	Exp. [ 23 ]	IBM-2	DDM	IBM-2	DDM
$0_2 \rightarrow 0_1$	0.7(0.1)	0.752	1.092	0.821	2.27
$2_2 \rightarrow 2_1$	4.5(0.5)	3.798	7.831	3.720	5.371
$4_2 \rightarrow 4_1$	6.6(2.10)	5.931	8.440	1.717	3.710

Table VIII  
Isomer shift  $\delta\langle r^2 \rangle$  for  $^{152-154}Sm$

$\delta\langle r^2 \rangle$	$^{152}Sm$			$^{154}Sm$		
	Exp. [ 28 ]	IBM-2	DDM	Exp. [ 28 ]	IB M-2	DD M
	>	18(4)*1 0 <sup>-2</sup>	16*1 0 <sup>-2</sup>	15.78*1 0 <sup>-2</sup>	1.2(0. 8)	2.51

#### IV. CONCLUSIONS

In the present work, the energies of low-lying levels, E2 and M1 reduced transition probabilities for  $^{152-154}Sm$  isotopes were calculated in the framework of IBM-2 and DDM. The calculated energy levels of low lying states were well reproduced, though some discrepancies remain, especially in the high spin states. The transition probability B(E2) between the ground band, the quasi gamma band and quasi beta band states is also calculated. Good agreement with the experimental energy for the low lying levels was obtained. The mixing ratios were also calculated after calculating E2 and M1 matrix elements. All the experimental and theoretical mixing ratios for  $^{152-154}Sm$  isotopes indicate a small M1 component which means that in the band-mixing transitions, M1 components are almost forbidden. However, an acceptable overall agreement between the experiment and theory was obtained as shown in table II.

The E0 quantities predicted by the two Models (IBM-2 and DDM) for  $0_3^+$  states were on the whole in poor agreement with experiment. Furthermore, the microscopic characters of the ground states and the excited  $0^+$  states in  $Sm^{152-154}$  would appear to be sufficiently complex to eliminate any description of them in terms of the simple model. We noticed that most of the experimental and calculated values for the  $X(E0/E2)$  ratio are small, which means that there is a small contribution of E0 transition on the life time of the  $0^+$  states. There are two high values of  $X(E0/E2)$  in transitions from  $0_2^+$  to  $0_1^+$  in  $^{152-154}Sm$  which means that this state decay mostly by the E0 is, the study of this state gives information about the shape of the nucleus, because

the E0 transitions matrix elements are connected strongly with the penetration of the atomic electron to the nucleus.

#### V. REFERENCES

- [1] N. N. Greenwood, A. Earnshaw, "Chemistry of elements", Program Press, Oxford, UK.(1984).
- [2] A.Passoja,J. Kantele, M.Luontama, R.Julin, E.Hammaren, P. Lipas and P. Toivonen, J. Phys. G: Nucl. Phys.12, 1047, (1986).
- [3] O. Scholten, F. Iachello and A. Arima, Ann. Phys. (N.Y) 115 (1978) 325.
- [4] M. A. Moinester, J. A. Alster, G. Azuelos, A. E. L. Dieperink, Nucl. Phys. A383, 264 (1982).
- [5] O. Castanos , P. Federman, A. Frank, S. Pittel, Nucl. Phys. A379 , 61 (1982).
- [6] S. M. Diab, Rom. Journ. Phys., Vol. 53, Nos. 3–4 (2008) 475.
- [7] F. Iachello and A. Arima, Ann. Phys, NY, 111, 253, (1976).
- [8] F. Iachello and A. Arima, Ann. Phys, NY, 111, 201, (1978).
- [9] F. Iachello and A. Arima, " The interacting Boson model", Cambridge University Press., (1987).
- [10] W. Pfeifer "An Interacting Boson IBM", Zurich. (1998).
- [11] D. S. Mosbah, J. A. Evans and W.D. Hamilton, J. Phys. G: Nucl. Phys. 20 (1994) 787.
- [12] K. Kumar and M. Baranger, Nucl. Phys. A122 (1968) 241.
- [13] K. Kumar and M. Baranger, Nucl. Phys. A122 (1968) 273.
- [14] K. Kumar, B. Remaud, P. Aguer, J.S. Vaagen, A.C . Rester, R. Foucher and J. H. Hamilton , Phys. Rev. C16 (1977) 1253.
- [15] K. Kumar, "Nuclear Models and the search for unity in nuclear physics" University of Bergen Press (1984).
- [16] *ENSDF*, [http:// www.nndc.bnl.gov/ensdf](http://www.nndc.bnl.gov/ensdf) (*Nuclear data sheet*) 2010
- [17] A . R. H. Subber, P. Park and W.D. Hamilton, J. Phys. G:Nucl. Phys.13, (1987) 161.
- [18] Y. Yamazaki, E. B. Shera, M. V. Hohen and R. M. Steffen, Phys. Rev. C 18 (1978) 147.
- [19] R. K. Smither, Phys. Rev. 150 (1960) 150.
- [20] N. Rud, G. T. Ewan, A. Christy, D. Ward, R. L. Graham and J. S. Geiger, Nucl. Phys. A191 (1972) 545.
- [21] Y. Yoshizawa, B. Elbek, B. Herskind and M.C. Olesen, Nucl. Phys 73 (1965) 273.
- [22] J. Lange, K. Kumar and J. H. Hamilton, Rev. of Mod. Phys., 54 (1982)119.
- [23] M. Sambataro, O. Scholten, A.E.L. Dieperink and G. Piccitto, Nucl. Phys.,A423 (1984) 333.
- [24] E.L. Church and J. Weneser, Phys. Rev., 103 (1956) 1035.
- [25] J.O. Rasmuesen, Nucl. Phys. 19 (1960) 85.
- [26] Philip D.. Duval and Bruce R. Barret, Phys. Rev. C23 (1981) 492.
- [27] R. Bijker, A.E.L. Dieperink, O.Scholten and R. Spanhoff, Nucl. Phys. A344 (1980) 207.
- [28] J.G. England, I. S. Grant, J.A.R. Griffith, D.E. Evans and P.M. Walker, J. Phys. G: Nucl. Part. Phys., 16 9190) 105.