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Similarity Solution for MHD Stagnation Point Flow and Heat Transfer over a Non-Linear Stretching Sheet

R. N. Jat[1], Abhishek Neemawat[2]
[1]Department of Mathematics, University of Rajasthan, Jaipur-302004, India
[2]Department of Mathematics, Yagyavalkya Institute of Technology, Jaipur-302022, India
E-mail: abhishekneemawat@yahoo.com

***Abstract* - A steady two-dimensional magneto-hydro-dynamic (MHD) stagnation-point flow of a viscous and electrically conducting fluid in the presence of transverse magnetic field towards a non-linearly stretching/shrinking sheet is studied. The stretching velocity and the external flow velocity impinges normal to the stretching/shrinking sheet are assumed to be in the form $U \sim x^m$, where m is a constant and x is the distance from the stagnation point. By using similarity transformations, the governing partial differential equations are converted into ordinary differential equations and solved by standard numerical techniques. The physical quantities of interest like skin-friction coefficient and the heat transfer rate at the surface with the governing parameters are tabulated and plotted. It is found that the solutions for a shrinking sheet are non-unique for $m > \frac{1}{3}$.**

***Keyword* - MHD boundary layer, dual solutions, stagnation point, similarity solution, stretching sheet.**

I. INTRODUCTION

Flow of an incompressible viscous fluid and heat transfer phenomenon over a stretching sheet has received great attention during the last decades owing to the abundance of practical applications in chemical and manufacturing processes, such as aerodynamic extrusion of plastic sheets, continuous casting of metals, glass fibre and paper production, cooling of metallic sheets or electronic chips and many others. In all these cases, the final product of desired characteristics depends on the rate of cooling and the process of stretching.

The problem of laminar boundary layer flow resulting from the flow of an incompressible viscous fluid past a stretching sheet where the velocity near the stagnation point is proportional to the distance from a stagnation point was considered by Crane [1]. Gupta and Gupta [2] examined the mass and heat transfer for the boundary layer over a stretching sheet subject to suction or blowing. The stability of such flow was shown by Bhattacaryya and Gupta [3]. Wang [4] obtained similarity solutions to the axisymmetric case. Mahapatra and Gupta [5] studied the problem by considering different stretching and free stream velocities. MHD boundary layer flow near the stagnation point of a linear stretching sheet has been studied by Jat and Chaudhary [6, 7]. MHD orthogonal stagnation point flow of a power-law fluid towards a stretching surface was investigated by Mahapatra et al. [8]. The unsteady problem of MHD-boundary layer flow over a linearly stretching surface with viscous dissipation at Joule heating was discussed by Jat and Chaudhary [9]. The radiation effects on the MHD flow over a linearly stretching sheet was studied by Jat and Chaudhary [10]. Lok et al. [11] extended Wang's [4] problem to MHD flow where dual solutions exist for small values of magnetic parameter. Bachok and Ishak [12] obtained similarity solutions for the stagnation point flow and heat transfer over a nonlinearly stretching/shrinking sheet.

In the present investigation we have studied the stagnation point flow toward a non-linearly stretching sheet with velocity $U_w = ax^m$ and free stream velocity $U_\infty = bx^m$, where a, b and m are constants. The object of the present paper is to study the flow and heat transfer for an electrically conducting fluid over a non-linear stretching sheet. Using the similarity transformations the governing boundary layer equations are reduced to ordinary two-point boundary value problem. Numerical solutions are obtained for momentum and heat equation and then discussed in detail the effects of various parameters on the velocity and temperature profile.

II. FORMULATION OF THE PROBLEM

Consider the two-dimensional steady flow of a viscous incompressible electrically conducting fluid near a stagnation point over a plane surface such that the sheet is stretched to its own plane with velocity proportional to some power of the distance from the stagnation point in the presence of an externally applied normal magnetic field of constant strength B_0. The induced magnetic field is assumed to be small compared to applied magnetic field and is neglected. The stretching surface has uniform temperature T_w and a non-linear velocity $U_w = ax^m$, while the velocity of the free stream (external flow) is $U_\infty = bx^m$. The fluid properties are assumed as constant. Under these assumptions, the simplified two-dimensional boundary layer equations governing the flow and heat transfer are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{...(1)}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_e B_0^2 u}{\rho} \qquad \text{...(2)}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma_e B_0^2 u^2}{\rho c_p} \qquad \text{...(3)}$$

The boundary conditions are:

$$y=0: u=U_w, v=0; \quad T=T_w$$

$$y\to\infty: u\to U_\infty; \qquad T\to T_\infty \qquad \text{...(4)}$$

where u and v are velocity components in x and y directions respectively, $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the density, μ is the coefficient of viscosity of the fluid, σ_e is the electrical conductivity of the fluid, T is the fluid temperature in the boundary layer, T_∞ is the free stream temperature, α is the thermal diffusivity, c_p is the specific heat at constant pressure.

III. ANALYSIS

The continuity equation (1) is identically satisfied by stream function $\psi(x, y)$ defined as

$$u=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x} \qquad \text{...(5)}$$

For the solution of momentum and energy equation (2) and (3), the following similarity transformations are defined:

$$\psi(x, y)=\sqrt{bx^{m+1}\nu}\, f(\eta) \qquad \text{...(6)}$$

$$\frac{T-T_\infty}{T_w-T_\infty}=\theta(\eta) \qquad \text{...(7)}$$

where $$\eta=y\sqrt{\frac{c}{\nu}} \qquad \text{...(8)}$$

thus we have

$$u=bx^m f'(\eta) \qquad \text{...(9)}$$

$$v=-\sqrt{b\nu}\left[\left(\frac{m+1}{2}\right)x^{\frac{m-1}{2}}f(\eta)+y\sqrt{\frac{b}{\nu}}\left(\frac{m-1}{2}\right)x^{m-1}f'(\eta)\right] \qquad \text{...(10)}$$

where prime denotes differentiation w.r.to η. With these similarity transformations, the equations (2) and (3) are reduced to the following non-linear ordinary differential equations with two-point boundary value problem:

$$f''' + \left(\frac{m+1}{2}\right) f f'' - m\left(1 - f'^2\right) - M f' = 0 \qquad ...(11)$$

$$\theta'' + \left(\frac{m+1}{2}\right) \Pr f \theta' + \Pr Ec f''^2 + M \Pr Ec f'^2 = 0$$

...(12)

The corresponding boundary conditions are:

$$\eta = 0 : f = 0, f' = \varepsilon; \quad \theta = 1$$

$$\eta \to \infty : f' \to 1; \qquad \theta \to 0 \qquad ...(13)$$

where $\varepsilon = \frac{a}{b}$ is the stretching parameter with $\varepsilon > 0$ for stretching and $\varepsilon < 0$ for shrinking and

$$M = \frac{\sigma_e B_0^2 x^{1-m}}{\rho b} \quad \text{(Magnetic Parameter)}$$

$$\Pr = \frac{\mu c_p}{\kappa} \quad \text{(Prandtl Number)}$$

$$Ec = \frac{U_\infty^2}{c_p (T_w - T_\infty)} \quad \text{(Eckert Number)} \qquad ...(14)$$

The physical quantities of interest of the problem are the skin-friction coefficient C_f and the Nusselt number Nu, can be expressed, respectively as

$$C_f = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U_w^2}{2}} = \frac{2}{\sqrt{\text{Re}}} f''(0) \qquad ...(15)$$

$$Nu = -\frac{x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_w - T_\infty)} = -\sqrt{\text{Re}}\,\theta'(0) \qquad ...(16)$$

where $\text{Re} = \frac{U_w x}{\nu}$ is the local Reylolds number.

IV. RESULTS AND DISCUSSIONS

Numerical solutions to the transformed ordinary differential equations (11) and (12) subject to the boundary condition (13) were obtained using Runge-Kutta method of fourth order with shooting technique. The equations were solved for several values of the stretching/shrinking parameter ε, the velocity exponent parameter m, the Prandtl number $\Pr$, magnetic parameter M and the Eckert number Ec. Figures 2 to 13 display the samples of velocity and temperature profiles for different values of m, $\Pr$, M, Ec and ε. These profiles satisfy the far field boundary conditions (13) asymptotically, which support the numerical results.

V. CONCLUSIONS

Similarity solutions for the problem of stagnation point flow towards a non-linearly stretching/shrinking sheet of electrically conducting fluid in the presence of magnetic field with constant surface temperature were investigated numerically. The governing partial differential equations were converted into ordinary differential equations by a similarity transformation, before being solved numerically by Runge-Kutta method of fourth order with shooting technique. The variations of the skin-friction coefficient and the heat transfer rate at the surface with the governing parameters were obtained and tabulated. Different from a stretching sheet, it was found that the solutions for a shrinking sheet are non-unique for $m > \frac{1}{3}$.

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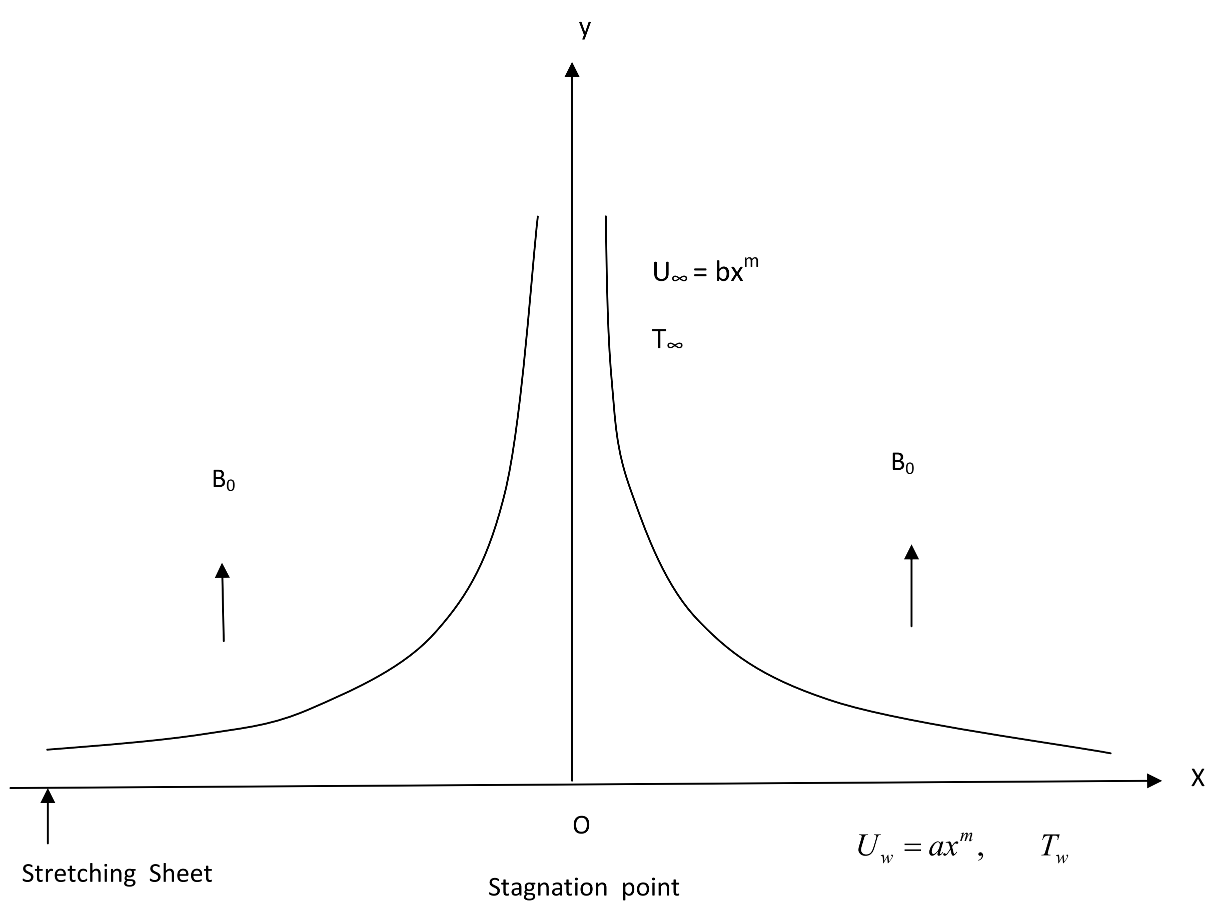

Fig. 1 Physical model and coordinate system

Table I: f''(0) for various values of ε when m = 2/3

M	ε = -1.1	ε = 0.0	ε = 1.1
0.0	0.821685	1.0226599	-0.1367212
0.1	0.745575	1.0859840	-0.0390835
0.2	0.642369	1.154705	0.0629987
0.3	_	1.229013	0.1696123
0.4	_	1.309052	0.2808254

Table II: f''(0) for various values of m when ε = 1.5

M	m = 0	m = 1/3	m = 2/3	m = 1
0	-0.32374	-0.5659	-0.73581	-0.87394
0.1	-0.07793	-0.4225	-0.62858	-0.78558
0.2	0.195523	-0.27041	-0.51711	-0.69466
0.3	0.496121	-0.10952	-0.40131	-0.60114
0.4	0.823054	0.060239	-0.28114	-0.50499

Table III: -θ' (0) for various values of ε when m = 2/3, Pr = 0.7 & Ec = 0.1

M	ε = -1.1	ε = 0.0	ε = 1.1
0.0	0.0583821	0.398663	0.604105
0.1	0.0510701	0.3960208	0.598490
0.2	0.0403331	0.391387	0.590596
0.3	_	_	0.5801732

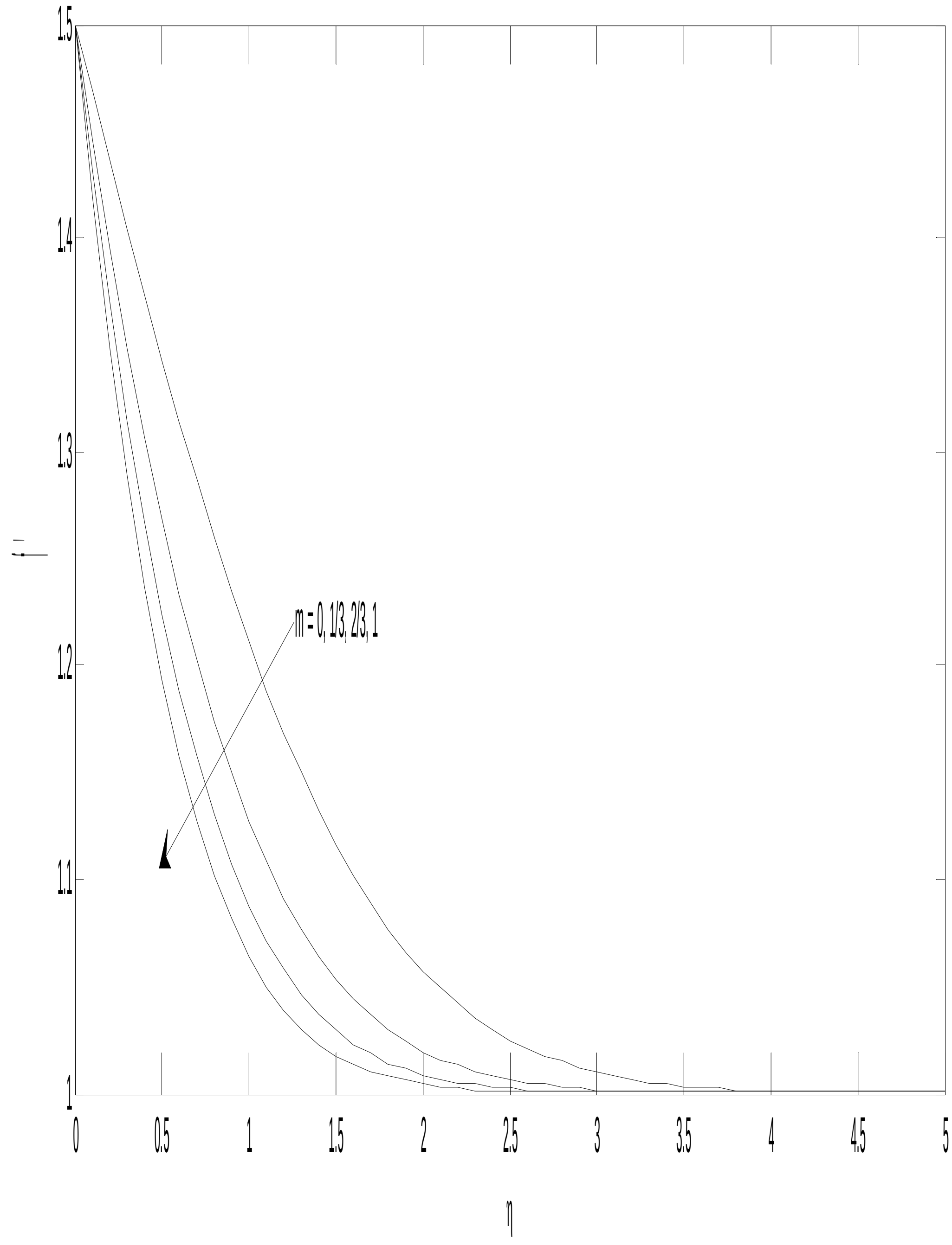

Fig. 2 Velocity profile against η for various values of m when ε = 1.5 & M = 0.0

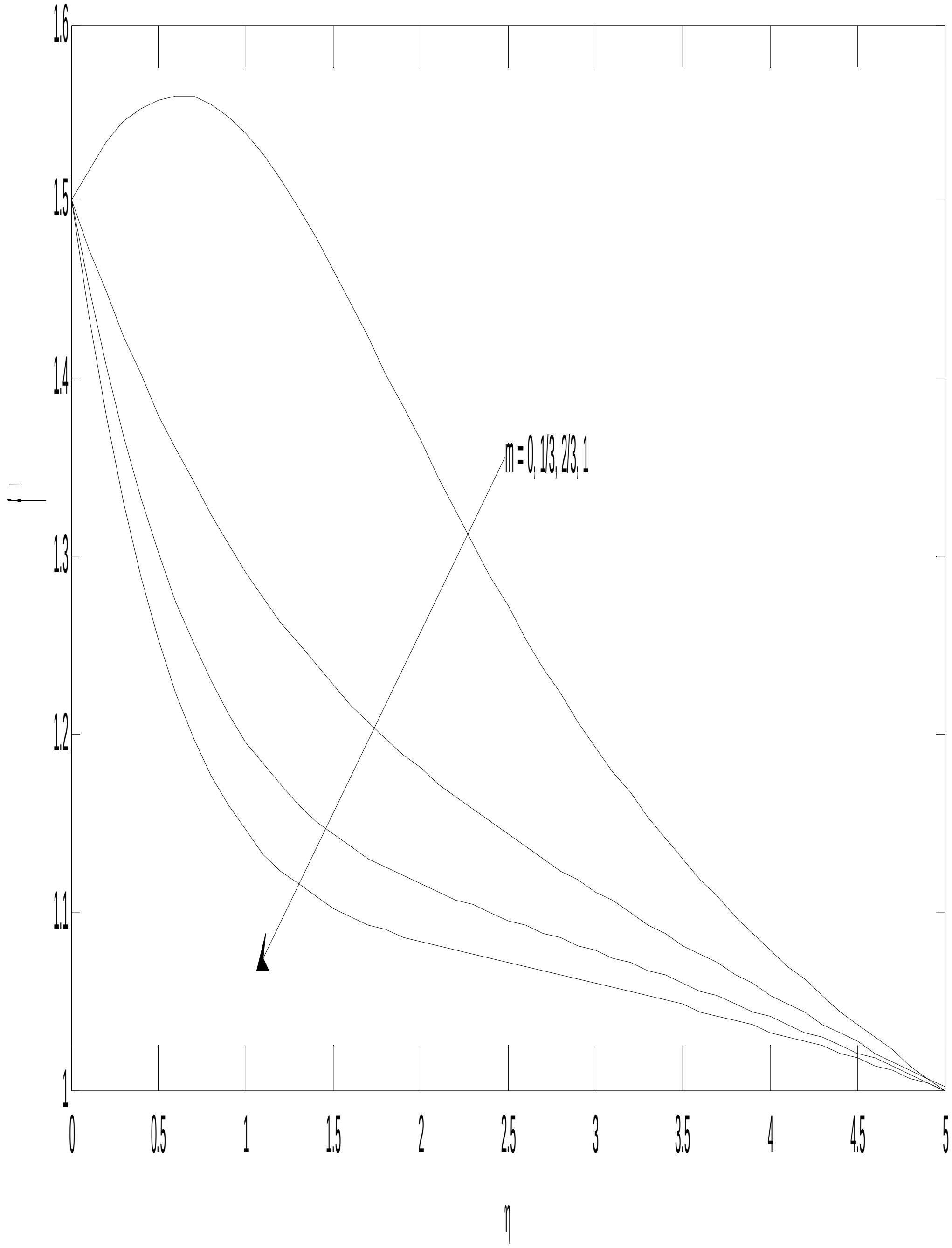

Fig. 3 Velocity profile against η for various values of m when ε = 1.5 & M = 0.2

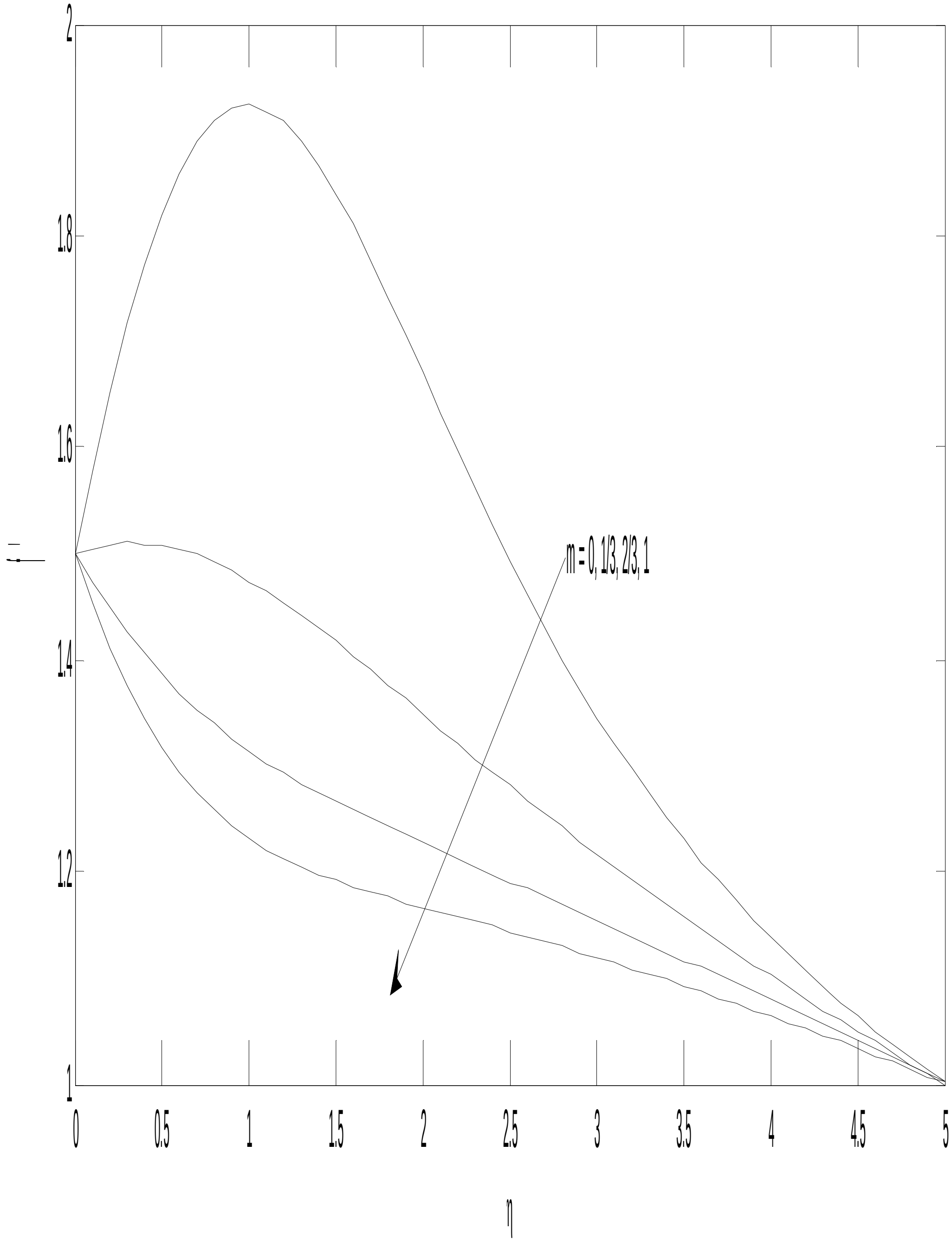

Fig. 4 Velocity profile against η for various values of m when ε = 1.5 & M = 0.4

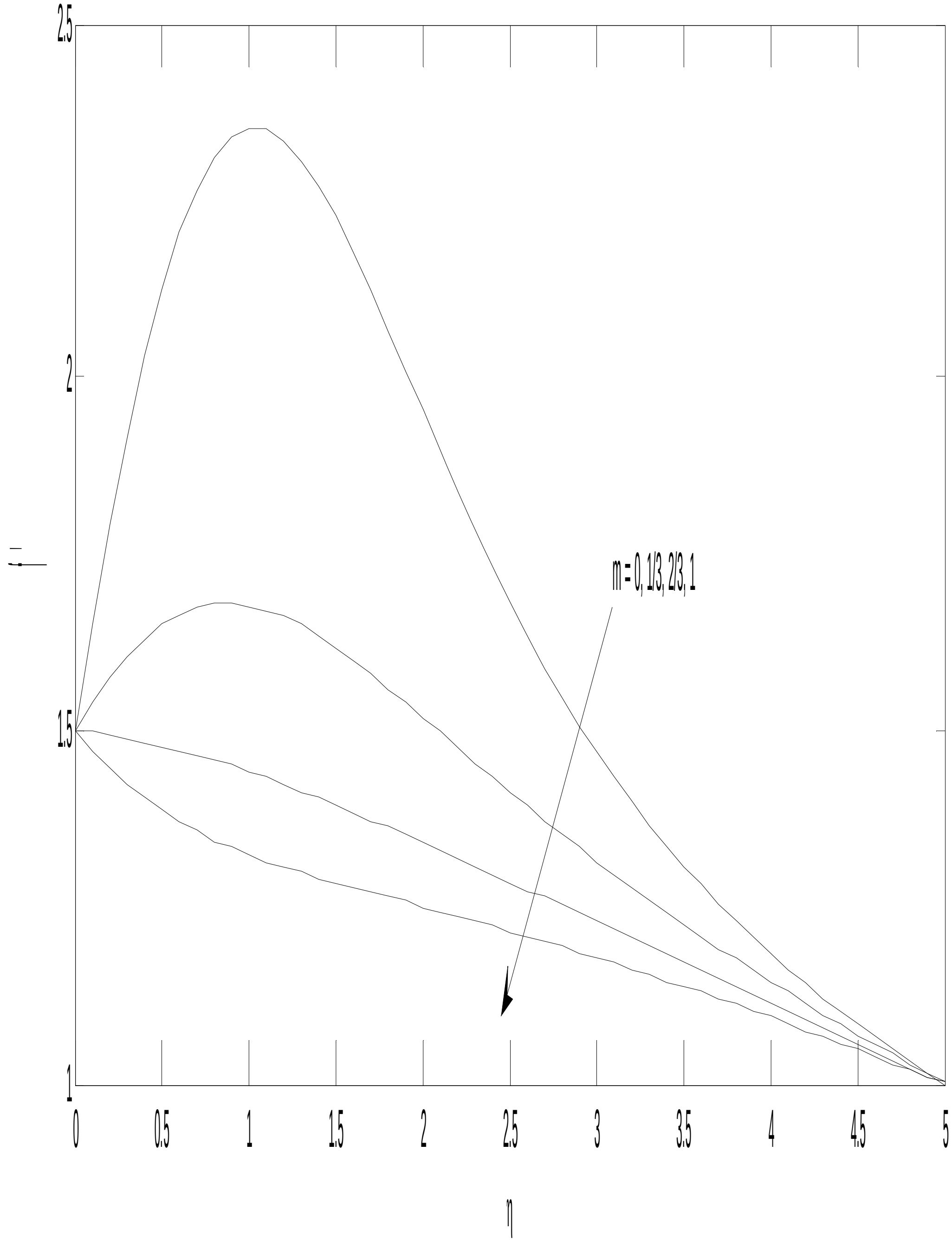

Fig. 5 Velocity profile against η for various values of m when ε = 1.5 & M = 0.6

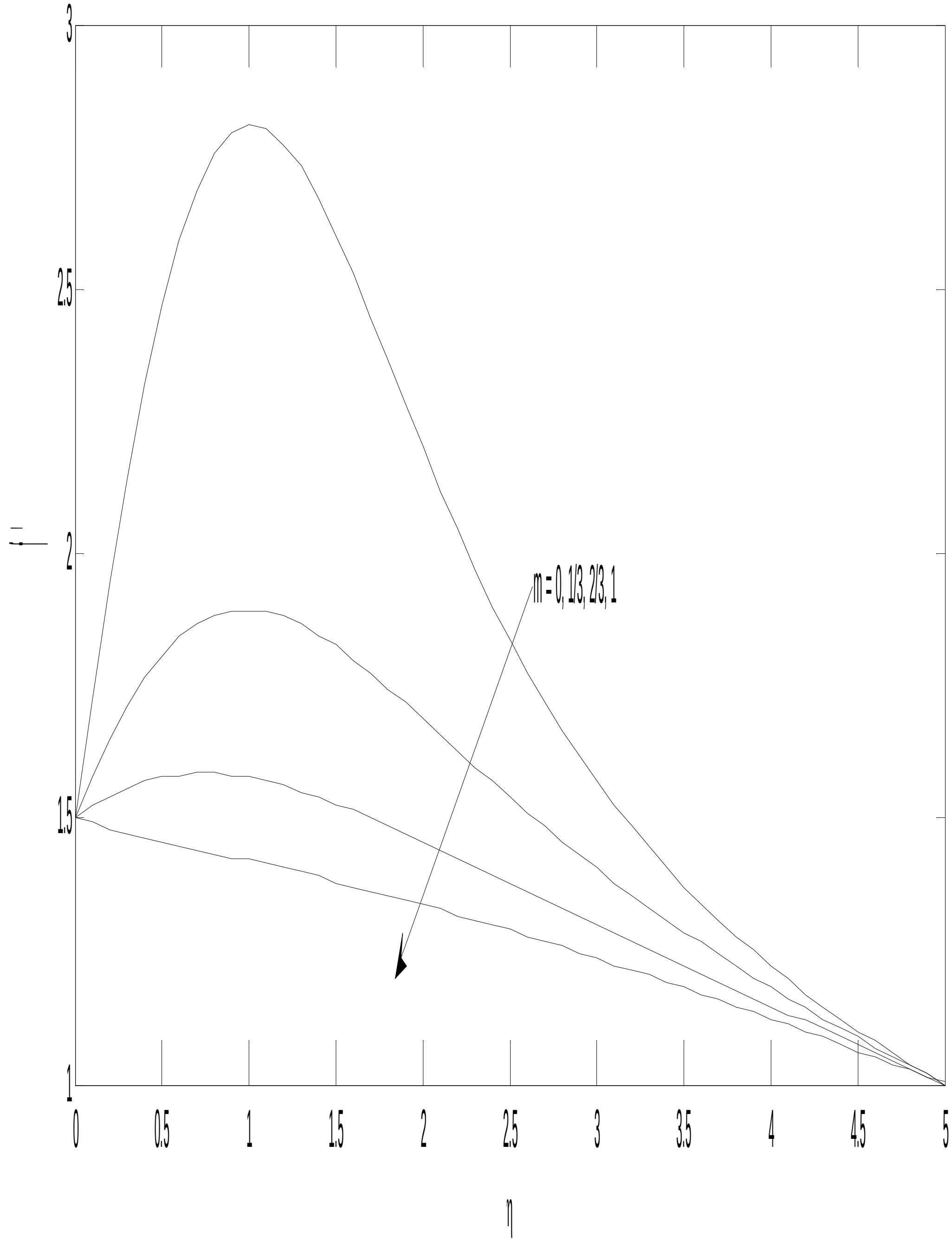

Fig. 6 Velocity profile against η for various values of m when ε = 1.5 & M = 0.8

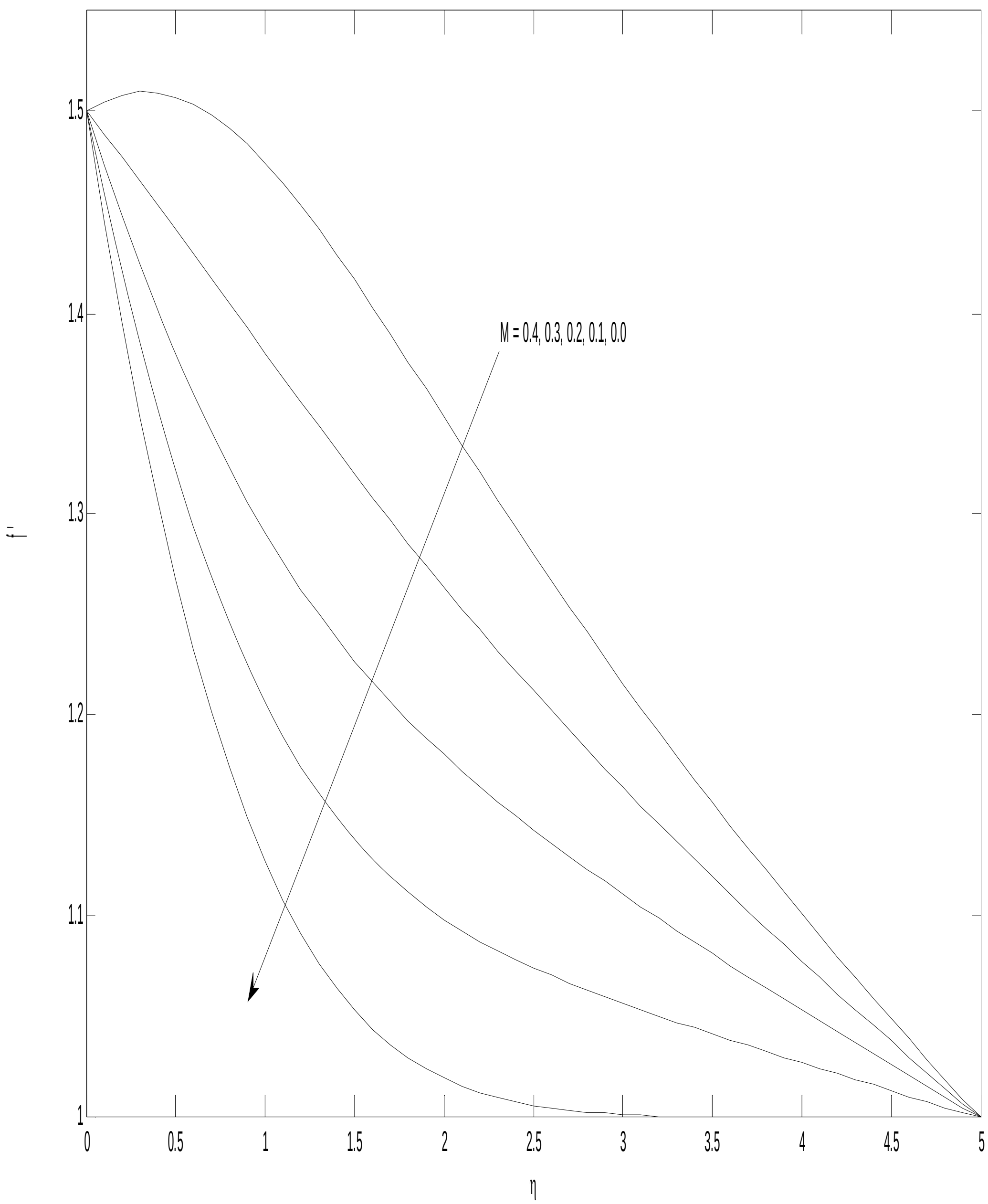

Fig. 7 Velocity profile against η for various values of M when ε = 1.5 & m = 1/3

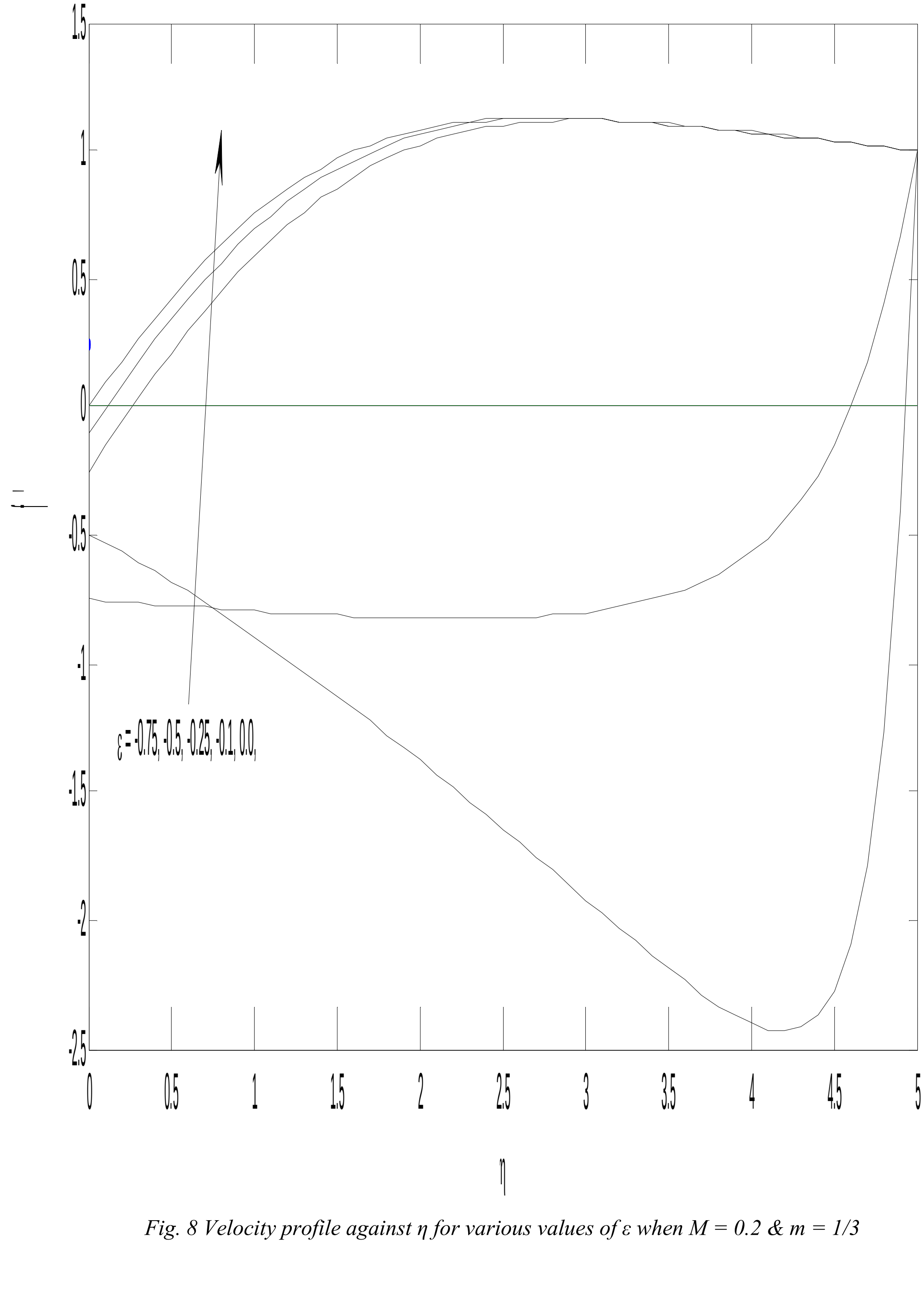

Fig. 8 Velocity profile against η for various values of ε when M = 0.2 & m = 1/3

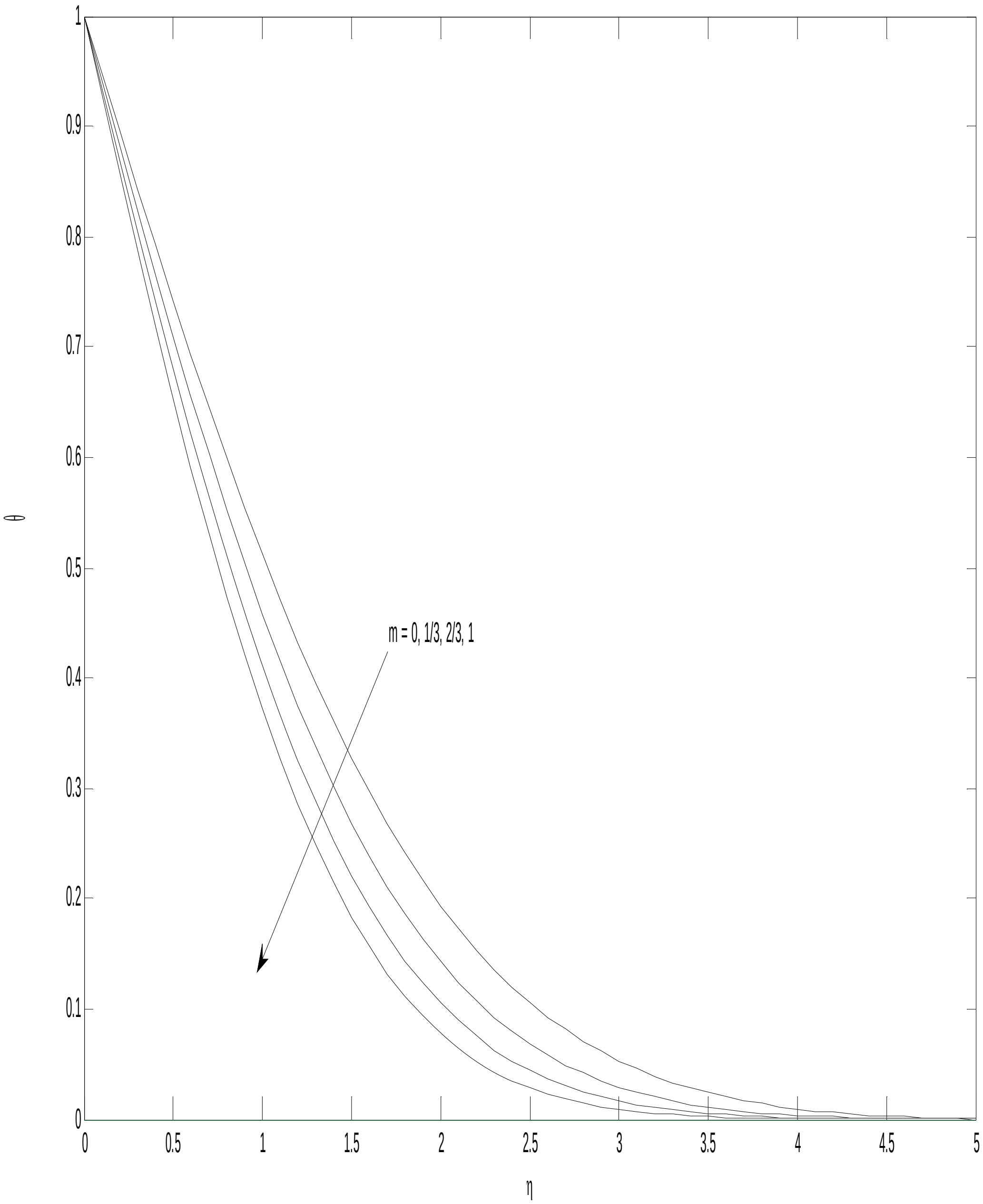

Fig. 9 Temperature profile against η for various values of m when ε = 1.5, M = 0, Pr = 0.7 & Ec = 0.0

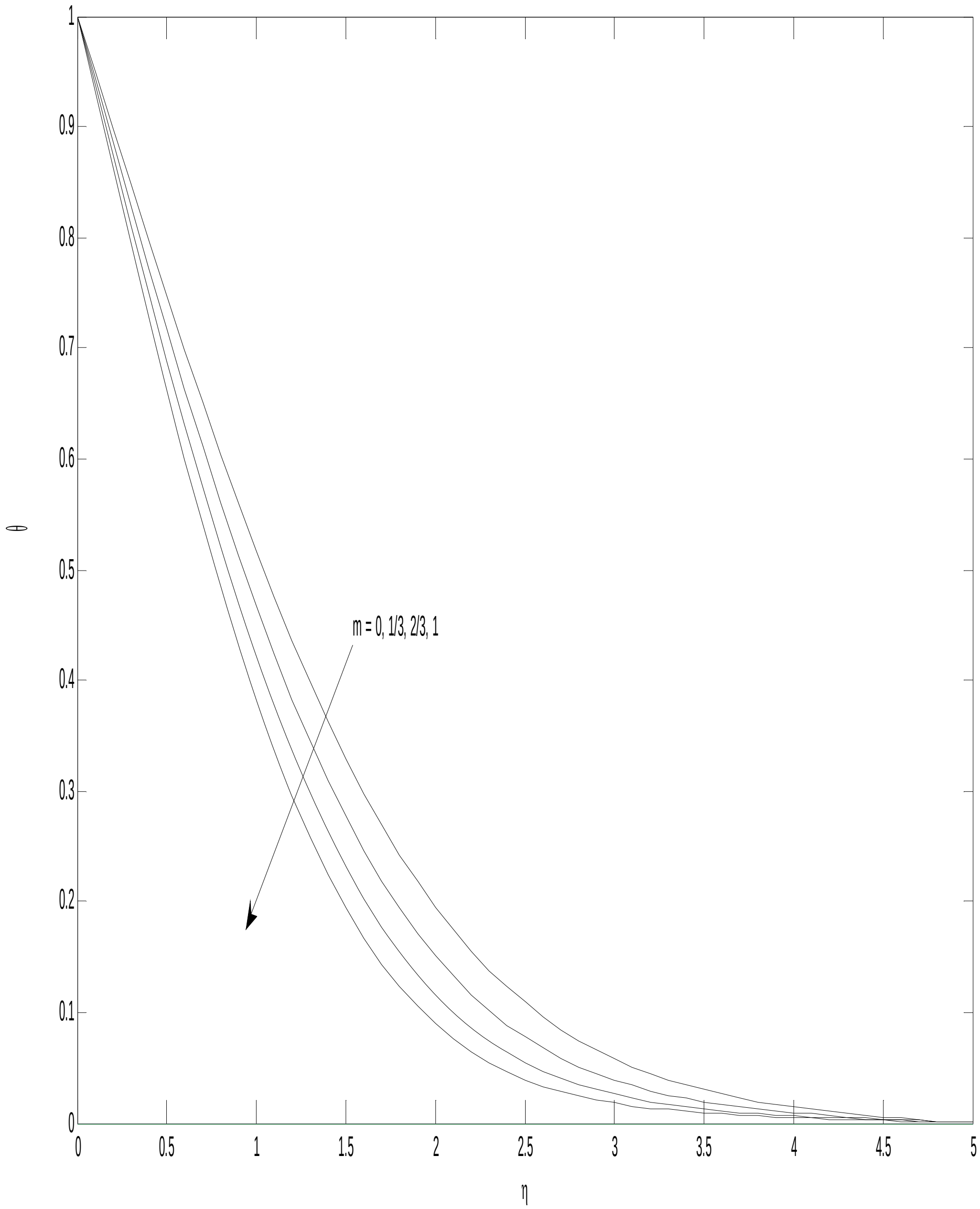

Fig. 10 Temperature profile against η for various values of m when ε = 1.5, M = 0.2, Pr = 0.7 & Ec = 0.1

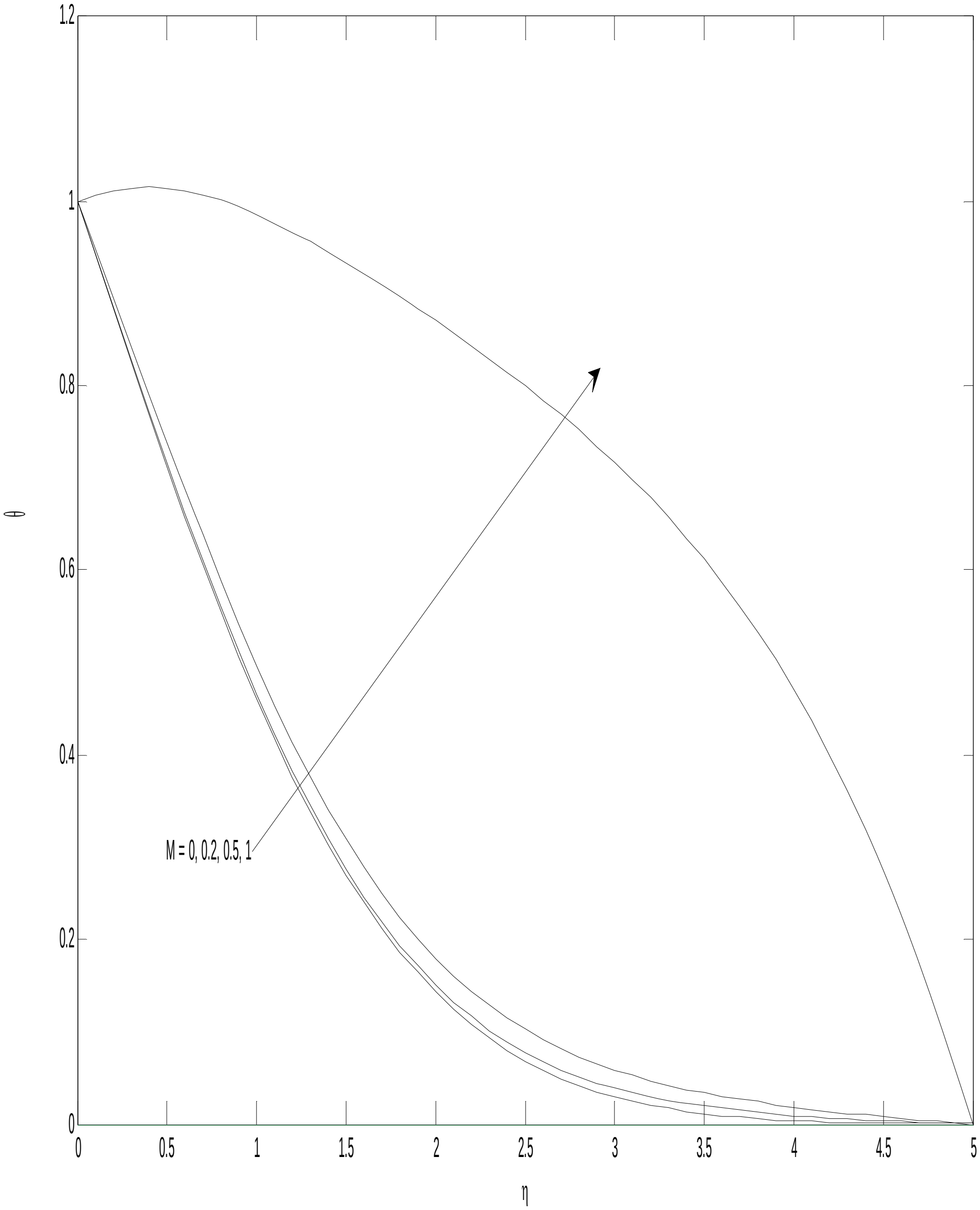

Fig. 11 Temperature profile against η for various values of M when ε = 1.5, m = 1/3, Pr = 0.7 & Ec = 0.1

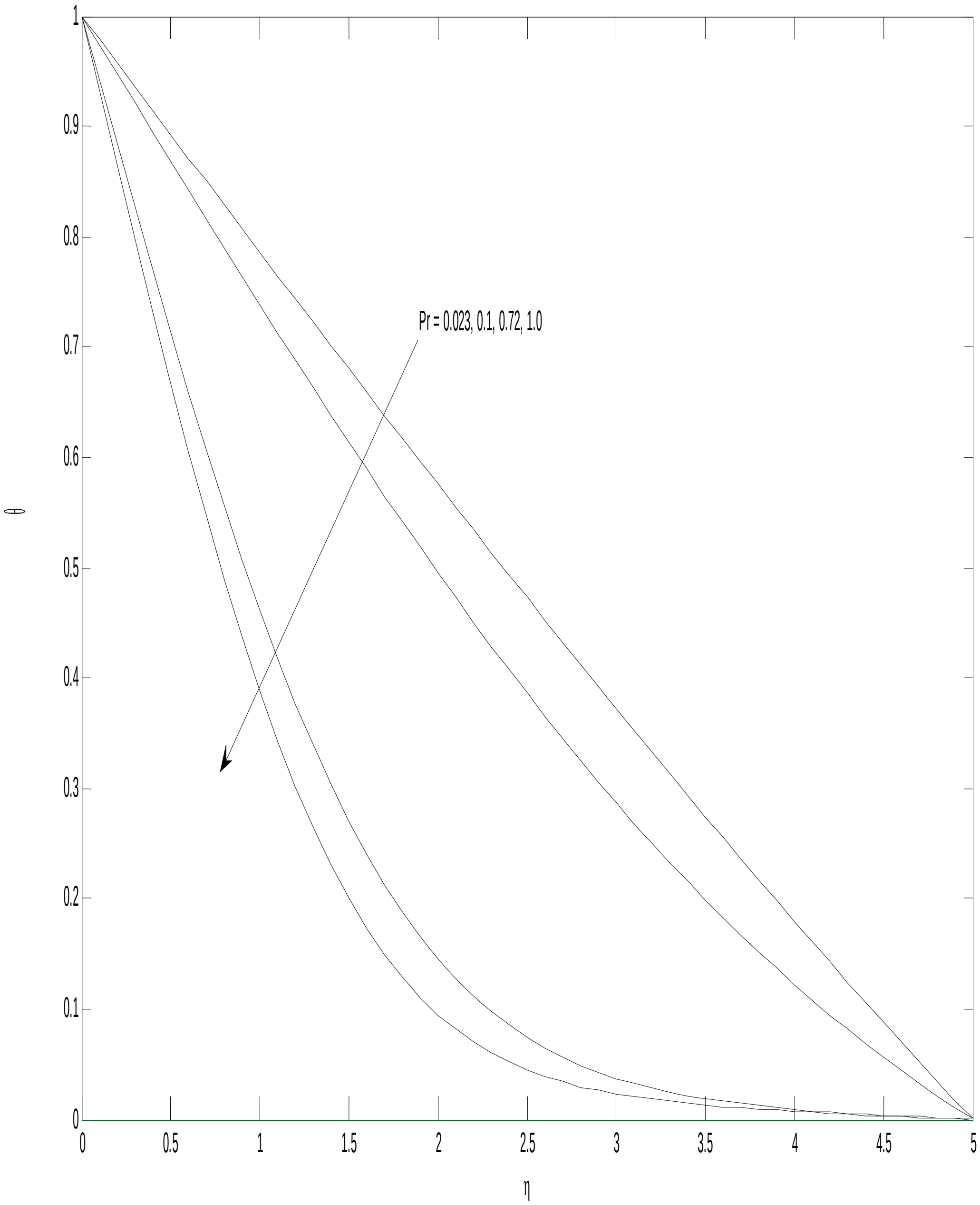

Fig. 12 Temperature profile against η for various values of Pr when ε = 1.5, m = 1/3, M = 0.2 & Ec = 0.1

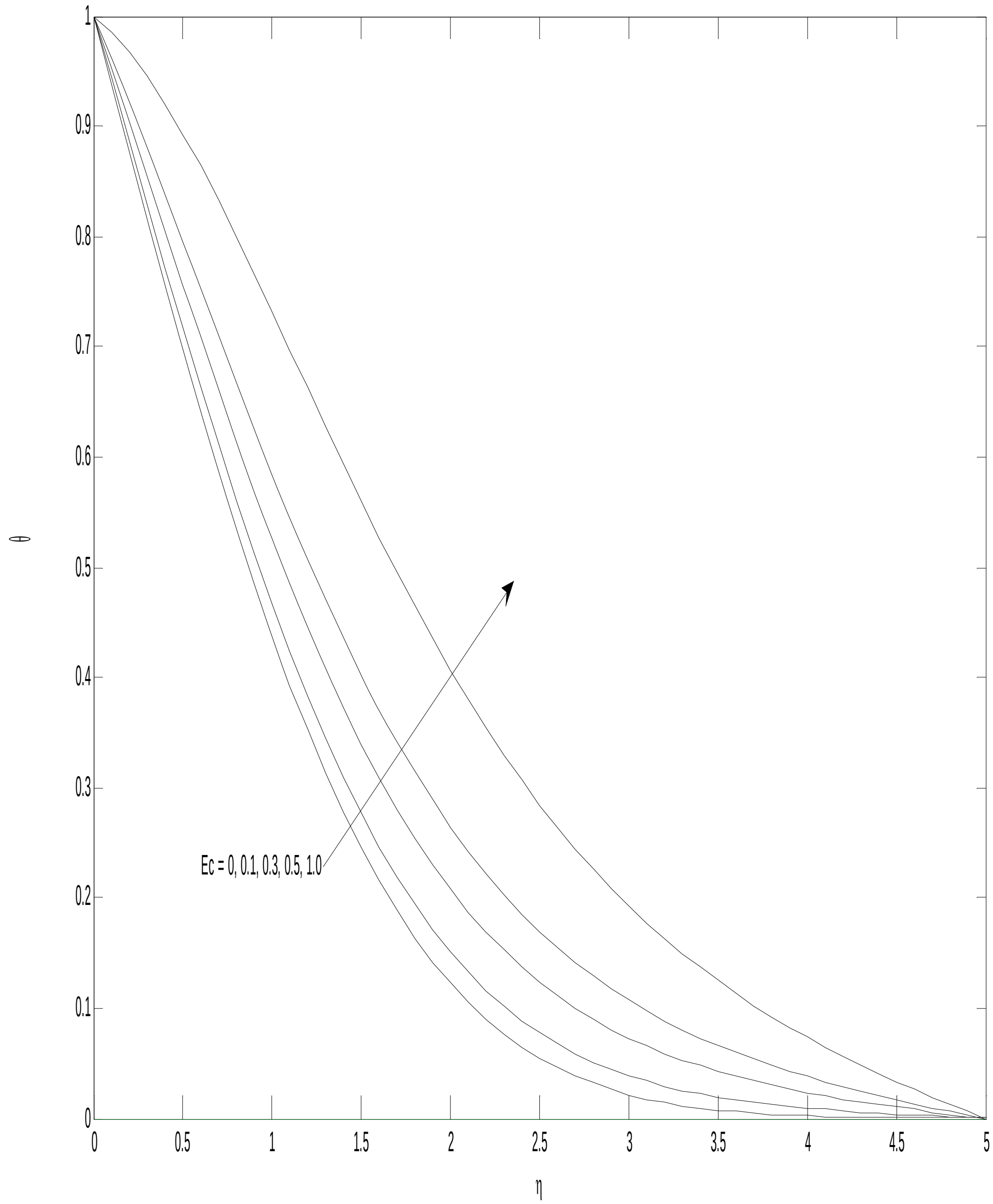

Fig. 13 Temperature profile against η for various values of Ec when ε = 1.5, m = 1/3, M = 0.2 & Pr = 0.7

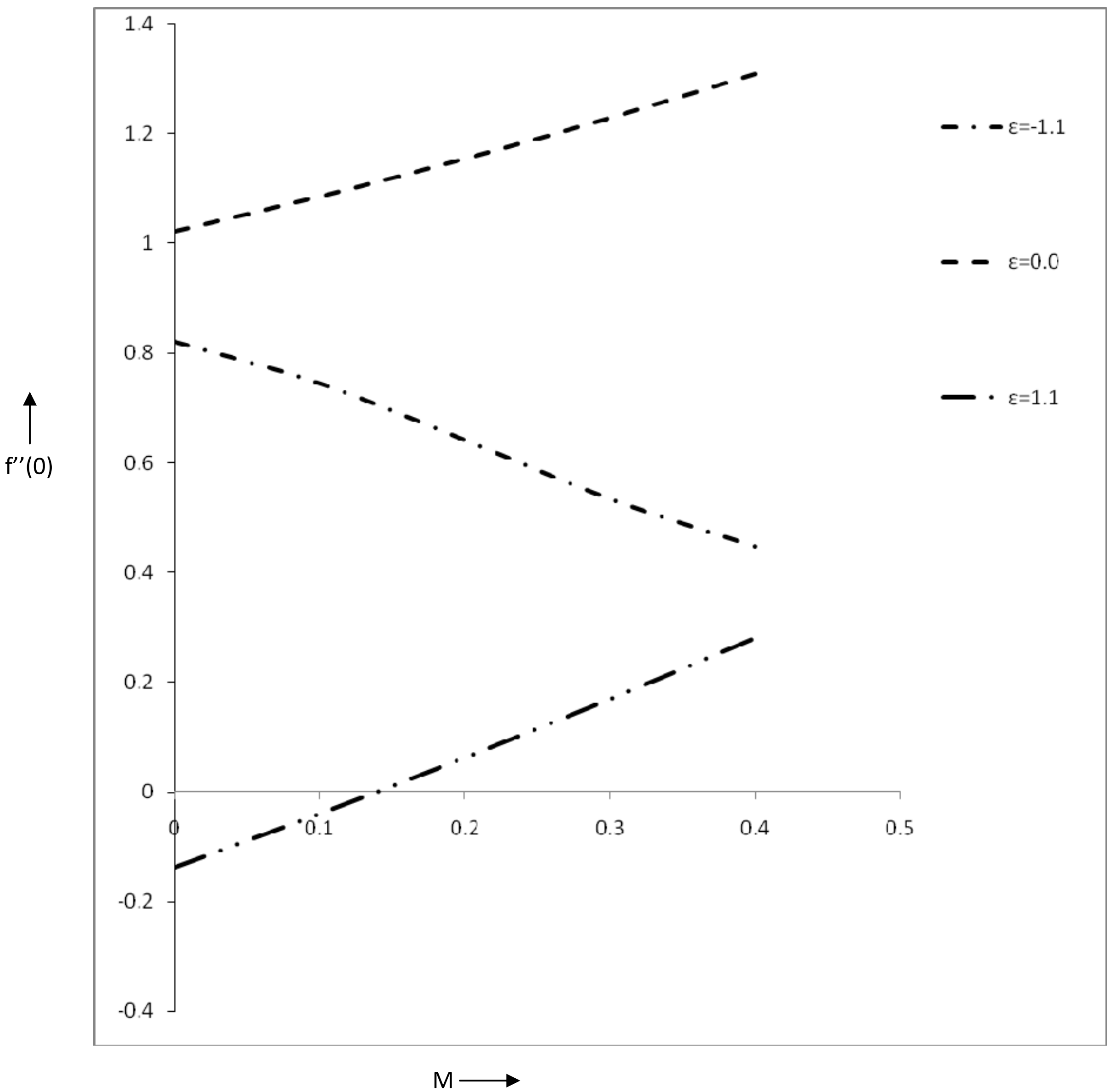

Fig. 14 Skin friction coefficient f''(0) as function of M for various values of ε when m = 2/3

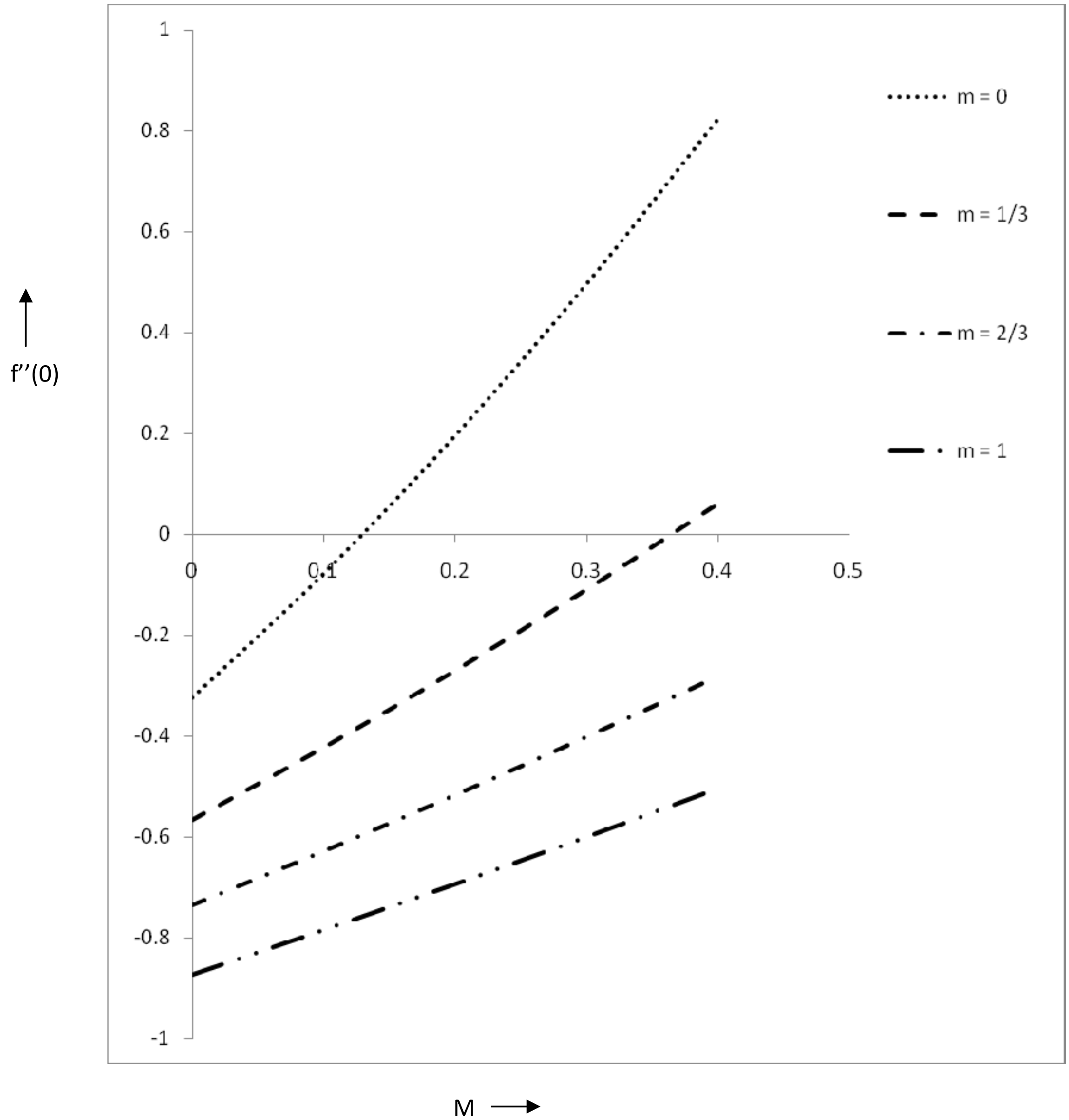

Fig. 15 Skin friction coefficient f''(0) as function of M for various values of m when ε = 1.5

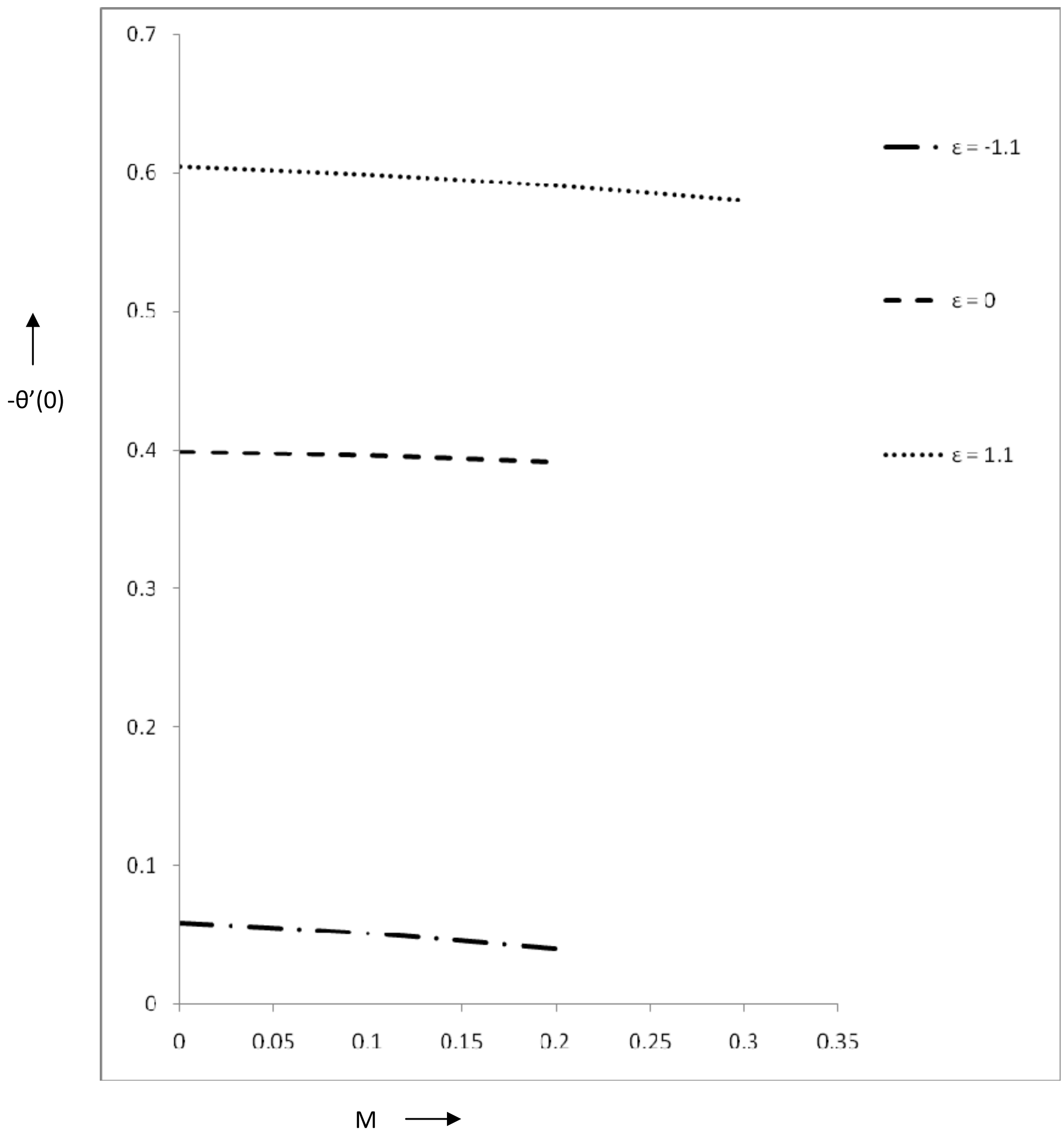

Fig. 16 Variation of –θ'(0) with M for various values of ε when m = 2/3, Pr = 0.7 & Ec = 0